PHYSICS ESSAYS **33**, 2 (2020)

Bekenstein–Hawking black hole entropy, Hawking temperature, and the Unruh effect: Insight from the laws of thermodynamics

Stephen J. Crothers[a] and Pierre-Marie Robitaille[b]
PO Box 1546, Sunshine Plaza 4558, QLD, Australia

(Received 28 January 2020; accepted 29 March 2020; published online 15 April 2020)

Abstract: The laws of thermodynamics play a central role in scientific inquiry, guiding physics as to the validity of hypothesized claims. It is for this reason that quantities of thermodynamic relevance must retain their character wherever they appear. Temperature, for example, must always be intensive, a requirement set by the 0th law. Otherwise, the very definition of temperature is compromised. Similarly, entropy must remain extensive, in order to conform to the second law. These rules must be observed whenever a system is large enough to be characterized by macroscopic quantities, such as volume or area. This explains why ensembles comprised of just a few atoms cannot be considered thermodynamic systems. In this regard, black holes are hypothesized to be large systems, characterized by the Schwarzschild radius ($r_s = 2GM/c^2$) and its associated "horizon" area ($A = 4\pi r_s^2$), where G, M, and c represent the universal constant of gravitation, the mass of the black hole, and the speed of light in vacuum, respectively. It can be readily demonstrated that Bekenstein–Hawking black hole entropy is nonextensive, while the Hawking and the Unruh temperatures are nonintensive. As a result, the associated equations violate the laws of thermodynamics and can hold no place in the physical sciences. *© 2020 Physics Essays Publication.* [http://dx.doi.org/10.4006/0836-1398-33.2.143]

Résumé: Les lois de la thermodynamique jouent un rôle central dans la recherche scientifique, guidant la physique quant à la validité de ses affirmations hypothétiques. C'est pour cette raison que les quantités qui ont de l'importance en thermodynamique doivent conserver leur caractère partout où elles apparaissent. La température, par exemple, doit toujours être intensive, une exigence fixée par le principe zéro de la thermodynamique. Sinon, la définition même de la température est compromise. De même, l'entropie doit rester extensive, afin de se conformer au deuxième principe de la thermodynamique. Ces règles doivent être respectées chaque fois qu'un système est suffisamment grand pour être caractérisé par des quantités macroscopiques, telles que le volume ou la surface. Cela explique pourquoi les ensembles composés de quelques atomes ne peuvent pas être considérés comme des systèmes thermodynamiques. À cet égard, les trous noirs sont supposés être de grands systèmes, caractérisés par le rayon de Schwarzschild ($r_s = 2GM/c^2$) et sa zone associée ($A = 4\pi r_s^2$), où G, M et c représentent la constante de gravitation universelle, la masse du trou noir et la vitesse de la lumière dans le vide, respectivement. Il peut être facilement démontré que l'entropie des trous noirs de Bekenstein-Hawking n'est pas extensive, tandis que les températures de Hawking et d'Unruh ne sont pas intensives. En conséquence, les équations associées violent les principe de la thermodynamique et ne peuvent tenir aucune place dans les sciences physiques.

Key words: Black Hole Entropy; Black Hole Temperature; Hawking Radiation; Black Hole Thermodynamics; Unruh Effect.

I. INTRODUCTION

Physics is an experimental science, which, over the span of nearly two centuries, has given birth and development to the laws of thermodynamics. Consequently, the variables required to describe any such system are also determined by experiment. Basic to the application of this branch of physics is the determination of relations between the coordinates of a system in thermodynamic equilibrium. Thermodynamic equations must be dimensionally balanced and, furthermore, they must also be thermodynamically balanced. Thus, if the thermodynamic character of one side of such an equation is intensive or extensive, then the other side must also be intensive or extensive, respectively. Any formulation involving thermodynamic coordinates that violates this balance is inadmissible. Canagaratna emphasized the need for thermodynamic balance by noting: "if one side of an equation is extensive (or intensive), then so must be the other side."[1] Landsberg further highlighted that "Its importance is such that it would be appropriate to regard it as a fourth 'law' of thermodynamics."[2]

As will been shown herein, it is readily apparent that the equations which describe black hole thermodynamics violate the rules that entropy must be expressed as an extensive property and temperature as an intensive property. Black hole entropy is not extensive. Black hole and Unruh temperatures are not intensive. As a result, they are completely detached from any link to thermodynamics. Perhaps, this

---

[a] sjcrothers@plasmaresources.com
[b] pierre@thermalphysics.org

explains why black hole proponents have strongly advocated the existence of black-hole thermodynamics. A note of caution is appropriate here, in that such theoretical pronouncements will always remain unsupported by experimental confirmation.

II. BEKENSTEIN–HAWKING BLACK HOLE ENTROPY

There are forces in the General Theory of Relativity, but gravity is not one of them, because it is spacetime curvature. Nevertheless, astrophysics utilizes Newtonian gravitational force for black holes by means of the Newtonian relation for escape speed[c]

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}}, \tag{1}$$

where G, M, and r correspond to the universal constant of gravitation, the black hole mass, and the radius, respectively. Setting $v_{\mathrm{esc}} = c$ and solving for the corresponding radius r_s gives

$$r_s = \frac{2GM}{c^2}, \tag{2}$$

the so-called "Schwarzschild radius" of a black hole, i.e., the radial distance from the black hole point-mass singularity to the "event horizon."

Bekenstein,[3,4] subsequently adopted by Hawking,[5,6] proposed that the area of a black hole event horizon constitutes the entropy of the black hole. It is common practice to present this mathematical relation in terms of "relativistic units." In order to avoid any uncertainty, the Bekenstein–Hawking black hole entropy equation in standard units is

$$S = \frac{\pi c^3 k_B}{2hG} A, \tag{3}$$

where S is entropy, c is the speed of light in vacuum, k_B is Boltzmann's constant, h is Planck's constant, G is the universal constant of gravitation, and A is the area of the event horizon. In the case of an uncharged, nonrotating black hole, $A = 4\pi r_s^2$, where r_s is given in Eq. (2). Equation (3) then becomes

$$S = \frac{8\pi^2 k_B G}{hc} M^2. \tag{4}$$

Although mass M is always extensive, M^2 is not, because extensive coordinates are additive.[7,8] The mass M of a system is the sum of the masses of its parts

$$M = \sum_{i=1}^{n} m_i = m_1 + m_2 + m_2 + \cdots + m_n.$$

Therefore,

$$M^2 = (m_1 + m_2 + m_3 + \cdots + m_n)^2 \neq m_1^2 + m_2^2 + m_3^2 + \cdots + m_n^2.$$

Thus, the squared mass of a system is not the sum of the squares of the masses of its parts, so mass squared is not extensive. Since entropy S is extensive, Eqs. (3) and (4) violate the laws of thermodynamics and are, therefore, inadmissible.[9]

Valid objections to Eq. (3) have also been advanced on the grounds that the black hole entropy is not a concave function,[10,11] thereby violating the second law. Although this is true, the simple thermodynamic imbalance of the black hole entropy equation is sufficient to prove it false. The proper conclusion is that black hole thermodynamics violates the laws of thermodynamics. One cannot argue that the hypothetical existence of black hole thermodynamics negates the experimentally determined laws of thermodynamics and that these laws must now be altered.

III. BLACK HOLE TEMPERATURE AND HAWKING RADIATION

According to modern theory, Hawking radiation can be emitted from a black hole. This radiation is thought to be a manifestation of a real thermodynamic process. The resulting temperature has always been viewed as a physical temperature, in accordance with the 0th law. However, it is readily proven that the concept of Hawking temperature violates the 0th law and the second law of thermodynamics. For an uncharged nonrotating black hole, Hawking radiation is said to correspond to a blackbody spectrum at a temperature T_H given by

$$T_H = \frac{\hbar c^3}{8\pi k_B G M}, \tag{5}$$

where $\hbar$ is the reduced Planck's constant, c is the speed of light in vacuum, G is the universal constant of gravitation, k_B is Boltzmann's constant, and M is the mass of the black hole. Equation (5) is etched into the gravestone of Stephen Hawking in Westminster Abbey.[12,13] However, temperature is an intensive property so it cannot be made to depend on the mass of a system, an extensive property, without an associated extensive property, like volume, which in combination with M leads to an intensive property. The left side of Eq. (5) is intensive but the right side is not, because it varies as $1/M$, all other terms being constants and unable to contribute to thermodynamic character.[8] The equation for Hawking temperature violates the laws of thermodynamics and is, therefore, invalid.[14]

Furthermore, the production of a blackbody spectrum depends absolutely on the presence of a physical vibrational lattice, as is well-known throughout metrology. The idea that such a blackbody spectrum can be generated from thermal equilibrium considerations alone is false.[15,16] As a result, black holes cannot be reconciled with the known laws of thermodynamics and Hawking radiation does not exist.

Astrophysics assigns Hawking temperature to more complex alleged black holes. In the case of the Kerr–Newman black hole (i.e., a charged and rotating black hole), the Hawking temperature is given by

---

[c] The theoretical Michell-Laplace dark body is not a black hole.

$$T_H = \frac{\hbar c\sqrt{\frac{G^2M^2}{c^4} - \frac{J^2}{M^2c^2} - \frac{Gq^2}{4\pi\varepsilon_o c^4}}}{4\pi k_B\left[\frac{GM}{c^2}\left(\frac{GM}{c^2} + \sqrt{\frac{G^2M^2}{c^4} - \frac{J^2}{M^2c^2} - \frac{Gq^2}{4\pi\varepsilon_o c^4}}\right) - \frac{Gq^2}{8\pi\varepsilon_o c^4}\right]}, \tag{6}$$

where J is angular momentum, and q is electric charge. If $q = 0$, The Hawking temperature of the Kerr black hole is

$$T_H = \frac{\hbar c\sqrt{\frac{G^2M^2}{c^4} - \frac{J^2}{M^2c^2}}}{4\pi k_B\left[\frac{GM}{c^2}\left(\frac{GM}{c^2} + \sqrt{\frac{G^2M^2}{c^4} - \frac{J^2}{M^2c^2}}\right)\right]}. \tag{7}$$

If $J = 0$, the Hawking temperature of the Reissner–Nordström black hole is

$$T_H = \frac{\hbar c\sqrt{\frac{G^2M^2}{c^4} - \frac{Gq^2}{4\pi\varepsilon_o c^4}}}{4\pi k_B\left[\frac{GM}{c^2}\left(\frac{GM}{c^2} + \sqrt{\frac{G^2M^2}{c^4} - \frac{Gq^2}{4\pi\varepsilon_o c^4}}\right) - \frac{Gq^2}{8\pi\varepsilon_o c^4}\right]}. \tag{8}$$

Finally, if $J = 0$ and $q = 0$, the Hawking temperature of the Schwarzschild black hole is

$$T_H = \frac{\hbar c\sqrt{\frac{G^2M^2}{c^4}}}{4\pi k_B\left[\frac{GM}{c^2}\left(\frac{GM}{c^2} + \sqrt{\frac{G^2M^2}{c^4}}\right)\right]} = \frac{\hbar c^3}{8\pi\, k_B GM}. \tag{9}$$

It is immediately apparent[8] that the right sides of Eqs. (6)–(9) are not intensive. Each of these equations is, therefore, invalid.

IV. THE UNRUH EFFECT

The arguments made relative to Hawking temperature can also be applied to the Unruh effect, as the two relations are essentially identical. According to this theoretical effect, "From the point of view of an accelerating observer or detector, empty space contains a gas of particles at a temperature proportional to the acceleration."[17] It is said that the "gas particles" are vacuum field quanta. Yet, on the other hand, "There is no need to talk about particles anywhere. Quantum field theory is about fields, not particles."[18]

In any case, the Newtonian relation for gravitational acceleration g is

$$g = \frac{GM}{r^2}. \tag{10}$$

According to the General Theory of Relativity, gravitation and acceleration are essentially indistinguishable, "… according to the equivalence principle, gravitation and acceleration are two sides of the same coin."[18] The uniform acceleration associated with the Unruh effect is linked accordingly to gravitational acceleration due to a black hole, "These results are independent of the means used to accelerate the detector, but depend only on the acceleration itself. … Applying these results on particle detectors to the black-hole evaporation problem, one finds that for a detector stationed near the horizon of the black hole, the transition probability of the detector per unit time can be calculated in a similar way to that for a static detector in Rindler coordinates."[19] "At a small distance (close to the black hole's event horizon, which is well defined without reference to accelerated worldlines), the thermal effects can, however, be attributed to the acceleration of the curves of constant Schwarzschild radial position, whereas a freely falling observer there sees, approximately, cold empty space (Unruh, 1977b; Fulling, 1977) This is the origin of the thermal emission or ambience, as viewed from afar, of black holes, as already emphasized in Unruh's original paper (Unruh, 1976)."[17]

Comparison of the temperature equations for the Unruh effect and for a black hole clearly reveals the association

$$T_{\text{Unruh}} = \frac{ha}{4\pi^2 ck_B}, \qquad T_H = \frac{hg}{4\pi^2 ck_B}. \tag{11}$$

Both a and g are uniform accelerations, the latter due to gravity from the Newtonian relation Eq. (10) above. The acceleration g "is the gravitational acceleration at the surface of the black hole,"[20] which "results from the effect of the strong gravitation on the vacuum field,"[20] and can be obtained explicitly by substituting Eq. (2) into Eq. (10), thus

$$g = \frac{GM}{r_S^2} = \frac{c^4}{4GM}. \tag{12}$$

Putting Eq. (12) into the second of Eqs. (11) yields

$$T_H = \frac{\hbar c^3}{8\pi\, k_B GM}, \tag{13}$$

the familiar form of the Hawking black hole temperature equation.[18,20,21,23]

In both Eqs. (11) and (13), temperature must be intensive, as required by the laws of thermodynamics.[8] Mass, however, can never be intensive. All other terms in Eqs. (11)–(13) are universal constants or pure numbers. Accordingly, the accelerations a and g are not intensive. Thus, both Eqs. (11) violate the laws of thermodynamics.

In the case of Schwarzschild spacetime, local or proper acceleration a is given by[19,24,25]

$$a = \frac{GM}{r^2\sqrt{1-\dfrac{2GM}{c^2 r}}}, \tag{14}$$

which diverges as $r \rightarrow r_s = 2GM/c^2$. Astrophysics attempts to circumvent this problem by defining black hole "surface gravity" κ, employing the Newtonian relation Eq. (10), as $\kappa = GM/r_S^2 = c^4/4GM$ [see Eq. (12) above], so that,[d)] "The surface gravity (κ) of a Schwarzschild black hole is the magnitude of the 4-acceleration of a static observer at r^* as measured by a static observer at $r = \infty$."[25]

However, "a static observer at $r = \infty$," cannot observe or measure the black hole or its surface gravity.

Unruh[19] invokes Eq. (14) for local (or proper) black hole acceleration (he sets $G = 1$, $c = 1$, $k_B = 1$, and $\hbar = 1$ throughout) and concludes by means of the first of Eqs. (11) "that the temperature and number of detectable particles diverge as $R \rightarrow 2\,\mathrm{M}$ in precisely the same way as for an accelerating detector in flat space-time. In both cases the temperature diverges as $a/2\pi$."[19]

To clarify, the Unruh temperature is obtained by treating the vacuum of "flat space-time" as a quantum field to obtain a Planckian distribution of field quanta, from which a temperature is obtained, namely,

$$\frac{1}{\exp\left(\dfrac{2\pi\omega c}{a}\right) - 1}, \tag{15}$$

where a is acceleration, c is the speed of light in vacuum, and ω is the frequency of the "Rindler mode"[19,25] of flat space-time. Comparing Eq. (15) with Planck's equation for thermal spectra,

$$\frac{2\pi\omega c}{a} = \frac{h\nu}{k_B T}, \tag{16}$$

then solving for temperature,

$$T_{\mathrm{Unruh}} = \frac{ha}{4\pi^2 c k_B}, \tag{17}$$

which is the first of Eqs. (11). Setting a in Eq. (17) by means of Eq. (14) yields Unruh's divergent temperature in association with Hawking black hole temperature. An "observer" or "detector" located at some finite position $r > r_s$ undergoes an acceleration given in Eq. (14). An observer "at $r = \infty$" observes not the acceleration Eq. (14) but the Newtonian acceleration $a_\infty = GM/r^2$. Then as $r \rightarrow r_s$, Eq. (17) diverges by Eq. (14) and

$$a_\infty \rightarrow \frac{GM}{r_s^2} = \frac{c^4}{4GM}, \tag{18}$$

so that the temperature at the black hole "surface" is Hawking's black hole temperature Eq. (13).

Unruh's "number of detectable particles diverge"[19] as his $R \rightarrow r_s$ also constitutes divergent temperature because "The distribution of particle number corresponds to a temperature,"[17] the distribution purportedly being either Bose–Einstein or Fermi–Dirac.[20]

As with Hawking radiation, the Unruh temperature is claimed to be that reported from a blackbody spectrum produced by quantum vacuum particles, "When a detector, coupled to a relativistic quantum field in its vacuum state, is uniformly accelerated through Minkowski spacetime, with proper acceleration a, it registers a thermal black body radiation at temperature $T = (\hbar a/2\pi c k_B) \sim 10^{-19} a$. In other words, it detects a thermal bath of particles."[18]

However, a thermal spectrum can only be produced by a physical vibrational lattice.[15] Neither "a gas of particles at a temperature proportional to the acceleration"[17] nor "a thermal bath of particles,"[18] nor "a thermal bath of scalar photons,"[19] possess a lattice structure.

It is also worth mentioning that Minkowski spacetime (i.e., flat spacetime) is infinite in extent. Hence, for the Unruh effect, "everything happens as if it is coupled to an infinite thermal reservoir. … And since a small system coupled to an infinite thermal reservoir at thermal equilibrium, evolves to that equilibrium, the Unruh effect emerges."[18] However, all thermodynamic systems are finite in extent by definition.[7,26–29] There is no equation of state possible for or thermal equilibrium with "an infinite thermal reservoir." One cannot, for example, specify the volume, any unique pressure, the internal energy, the mass, any change in heat, or entropy of an infinite thermal reservoir.

Relative to what constitutes a thermodynamics system, the point has clearly been made, "The term system, as used in thermodynamics, refers to a definite quantity of matter bounded by some closed surface. The surface may be a real one … or it may be imaginary. … It is very important that the meaning of the term 'system' be kept clearly in mind. … Any systems which can interchange energy with a given system are called the surroundings of that system."[27] In this regard, it is evident, that an infinite thermal reservoir is not a thermodynamic system and, therefore, cannot be assigned a definite temperature. No thermodynamic system can be coupled to an "infinite thermal reservoir at thermal equilibrium" and "evolve" to that equilibrium, so that "the Unruh effect emerges." Thermal equilibrium is not defined for an infinite thermal reservoir.

Accelerations are manifestations of unbalanced forces.[e)] The Unruh temperature and the black hole temperature are directly proportional to acceleration—in the latter case by invoking Newtonian gravitational acceleration.

[d)] Here $r^* = r_s$.

[e)] Recall that there are no gravitational forces in General Relativity.

Accelerations are not thermodynamic coordinates. Inclusion of gravity for assignment of a temperature relation invariably leads to nonintensive temperature,[8,9,14,30–32] violating the 0th and second laws of thermodynamics. In this regard, it is interesting to note that the temperature of an ideal gas in thermal equilibrium in a container at rest cannot be influenced by the presence of a gravitational field, proven by L. Boltzmann[33] in 1896.

V. PHYSICS BY HYPOTHESIS

Confronted with violations of thermodynamics, astronomy and cosmology seek to disregard the laws of thermodynamics in order to simply permit theories that violate these laws. For example, in order to salvage black hole thermodynamics, they advance *ad hoc* new theoretically derived laws which can never be experimentally validated. The procedure is as follows, "Define the generalized entropy, S', to be the sum of the ordinary entropy, S, of matter outside a black hole plus the black hole entropy

$$S' \equiv S + S_{bh}.$$

Finally, replace the ordinary second law of thermodynamics by the generalized second law (GSL): The total generalized entropy of the universe never decreases with time"[34]

$$\Delta S' \geq 0.$$

Note that, not only is it proposed to move the second law of thermodynamics on the hypothesis of black holes, entropy is to also be applied to a space of infinite extent (the universe), i.e., infinite spatial extent is to become a thermodynamic system; provided a big bang universe with $k=1$ is excluded. However, if a big bang universe of $k=1$ is assumed, the universe is then of finite spacetime extent, whereas all black hole spacetimes are infinite in spacetime extent by mathematical construction. Furthermore, $S' \equiv S + S_{bh}$ is a statement that entropy is additive and, therefore, extensive, in accordance with the laws of thermodynamics, whereas the Bekenstein–Hawking black hole entropy S_{bh} is not additive and therefore not extensive. The generalized entropy is thus defined as additive but contains a component, S_{bh}, that is not additive; in other words, the generalized entropy is both extensive and not extensive. The "generalized entropy," S', and the Bekenstein–Hawking black hole entropy Eq. (3) are contradictory, and false.

Similarly, in terms of a "quantized vacuum field," an infinite spatial extent is to be regarded as a thermodynamic system, with a definite temperature: "We go to the limit where the volume of our quantization box becomes very large, $V \to \infty$, … Once one accepts the simplest features of a quantized vacuum field,"[20] the Unruh temperature equation "emerges as a consequence of time dependent Doppler shifts in the field seen by the accelerated observer."[20]

VI. CONCLUSIONS

Given that black hole entropy is not extensive, and that black hole temperature and Unruh temperature are not intensive, it is certain that these theoretical constructs stand in violation of the 0th and second laws of thermodynamics.

Hawking radiation and the Unruh effect are purportedly sources of blackbody spectra. However, since there is no physical vibrational lattice present in either case, they are deprived of the only means by which Planckian spectra can be generated.[15]

Gravity plays no role in thermodynamics. Infusing thermodynamics and the kinetic theory of gases with gravity by combining thermodynamic expressions with gravitational expressions produces violations of the laws of thermodynamics. "Gravitational thermodynamics" has no scientific basis.[32]

If physics is to continue making real progress relative to the understanding of nature, then the laws of thermodynamics must not be altered *ad libitum*, in order to permit theories that discount the intensive and extensive characters of temperature and entropy, respectively. The laws of thermodynamics, as determined by experiment on Earth, serve to guide theoretical investigation and modeling. This fundamental principle must be preserved and, therefore, Hawking radiation, the Unruh effect, and black holes, must not be allowed to be created merely from mathematics.

[1] S. G. Canagaratna, J. Chem. Educ. **69**, 95 (1992).
[2] P. T. Landsberg, "Monographs in statistical physics and thermodynamics," *Thermodynamics, with Quantum Statistical Illustrations*, Vol. 2 (Interscience Publishers, Inc., New York, 1961).
[3] J. D. Bekenstein, Lett. Nuovo Cimento **4**, 737 (1972).
[4] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1980).
[5] S. W. Hawking, Nature **248**, 30 (1974).
[6] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[7] A. B. Pippard, *The Elements of Classical Thermodynamics* (Cambridge University Press, Cambridge, UK, 1960).
[8] P.-M. Robitaille and S. J. Crothers, Phys. Essays **32**, 158 (2019).
[9] P.-M. Robitaille, *Black Hole Entropy: A Violation of the Second Law of Thermodynamics* (APS NWS, Toledo, OH, 2018), http://meetings.aps.org/Meeting/OSS18/Session/G2.4.
[10] B. H. Lavenda and J. Dunning-Davies, Class. Quantum Grav. **5**, 149 (1988).
[11] B. H. Lavenda and J. Dunning-Davies, Found. Phys. Lett. **3**, 435 (1990).
[12] See https://www.westminster-abbey.org/abbey-commemorations/commemorations/stephen-hawking for an image of Stephen Hawking's gravestone with his black hole temperature equation.
[13] See https://stephenhawkingfoundation.org for an audio-visual presentation containing the Bekenstein-Hawking black hole entropy equation; https://stephenhawkingfoundation.org/medallion/ for the black hole temperature equation on a medallion.
[14] P.-M. Robitaille, *Hawking Radiation: A Violation of the Zeroth Law of Thermodynamics* (APS NES, Boston, MA, 2018), http://meetings.aps.org/Meeting/NES18/Session/D01.3.
[15] P.-M. Robitaille, IEEE Trans. Plasma Sci. **31**, 1263 (2003).
[16] P.-M. Robitaille and S. J. Crothers, Prog. Phys. **11**, 120 (2015); e-print http://www.ptep-online.com/2015/PP-41-04.PDF.
[17] S. A. Fulling and G. E. A. Matsas, Scholarpedia **9**, 31789 (2014); e-print http://www.scholarpedia.org/article/Unruh_effect.
[18] S. De Bièvre and M. Merkli, Class. Quantum Grav. **23**, 1 (2006); e-print http://math.univ-lille1.fr/~debievre/Talks/unruhrolla09A.pdf.
[19] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[20] P. M. Alsing and P. W. Milonni, Am. J. Phys. **72**, 1524 (2004).
[21] R. M. Wald, *General Relativity* (The University of Chicago Press, Chicago, IL, 1984).
[22] R. A. Mould, *Basic Relativity* (Springer-Verlag, New York, 1994).
[23] B. Schutz, *A First Course in General Relativity* (Cambridge University Press, Cambridge, UK, 2009).
[24] N. Doughty, Am. J. Phys. **49**, 720 (1981).

[25] M. Socolovsky, Ann. Fond. Louis Broglie **39**, 1 (2014).
[26] M. W. Zemansky and R. H. Dittman, *Heat and Thermodynamics* (McGraw-Hill Book Company, New York, 1981).
[27] F. W. Sears, *An Introduction to Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics* (Addison-Wesley Publishing Company, Inc., Boston, 1955).
[28] C. McMullen, *A Guide to Thermal Physics from the Fundamentals thru Callen-Level Equilibrium Thermodynamics* (CreateSpace, Chennai, 2010), ISBN: 1453772804.
[29] B. V. Karlekar, *Thermodynamics for Engineers* (Prentice-Hall, Inc., New Jersey, 1983).
[30] P.-M. Robitaille, Prog. Phys. 14, 179, (2018); e-print http://ptep-online.com/2018/PP-54-13.PDF.
[31] P.-M. Robitaille, *Does Gravitational Collapse Occur? Insight from the Laws of Thermodynamics!* (Sky Scholar, Ohio, 2018), https://www.youtube.com/watch?v=DtMee3rrHDY.
[32] P.-M. Robitaille, *Gravitational Thermodynamics – Is it Science*? (Sky Scholar, Ohio, 2018), https://www.youtube.com/watch?v=mZ2F2Kw5-nQ.
[33] L. Boltzmann, *Lectures on Gas Theory*, translated by S. G. Brush (University of California Press, Berkeley, CA, 1964), pp. 141–143.
[34] R. M. Wald, Living Rev. Relativity **4**, 6 (2001), DOI: https://doi.org/10.12942/lrr-2001-6; e-print: ArXiv: Gr-qc/9912119v2 2000.